

Pseudogap of Color Superconductivity

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We show that the pseudogap of the quark density of states is formed in hot quark matter as a precursory phenomenon of the color superconductivity on the basis of a low-energy effective theory. We clarify that the soft mode of the di-quark pair field gives rise to a peculiar behavior of the quark dispersion relation and a short life-time of the quasiparticles near the Fermi surface, both of which make a depression of the density of states of quarks. Our result suggests that the appearance of the pseudogap is a universal phenomenon of strong coupling superconductors, irrespective of the dimensionality.

§1. Introduction

It is one of the central issues in hadron physics to determine the phase structure of QCD at large chemical potential μ and relatively low temperature T . The recent renewed interest in the color superconductivity (CS)¹⁾ stimulated intensive studies in these region, which in turn are revealing rich physics of the high density hadron/quark matter with CS.²⁾

Possible physical realizations of the CS in compact stars or ultrarelativistic heavy-ion collisions are also discussed actively. Here, note that these systems are at relatively low density ρ where the strong coupling nature of QCD may show up. The strong coupling may invalidate the mean-field approximation à la BCS theory,¹⁾ and make the so-called Ginzburg region so wide that precursory fluctuations of the pair field can have a prominent strength and may give rise to physically significant effects even above the critical temperature T_c .³⁾

The existence of the large fluctuations suggests us that the CS may share some basic properties with the high- T_c superconductivity (HTSC) of cuprates rather than with the usual superconductivity in metals. One of the most characteristic phenomena of HTSC is the existence of the *pseudogap*, i.e., the anomalous depression of density of state (DOS) $N(\omega)$ as a function of the fermion energy ω around the Fermi surface above T_c . Although the mechanism of the pseudogap in HTSC is still controversial, precursory fluctuations of the pair field seem to be basic ingredients to realize the pseudogap.⁴⁾ Thus, one may naturally expect that the pseudogap of the quark density of states exists as a precursory phenomenon of the CS at finite T . In this talk, we shall show that it is the case using a chiral model.⁵⁾

§2. Formalism

To describe a system at relatively low T and ρ , it is appropriate to adopt a low-energy effective theory of QCD. Here we employ the Nambu-Jona-Lasinio model with the scalar-diquark interaction in the chiral limit,

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + G_C \sum_A (\bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi^C) (\bar{\psi}^C i \gamma_5 \tau_2 \lambda_A \psi) + G_S [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \vec{\tau} \psi)^2], \quad (2.1)$$

where $\psi^C \equiv C \bar{\psi}^T$, with $C = i \gamma_2 \gamma_0$. Here, τ_2 and λ_A mean the antisymmetric flavor SU(2) and color SU(3) matrices, respectively. The coupling G_S and the three dimensional momentum cutoff $\Lambda = 650$ MeV are determined so as to reproduce the physical quantities and we choose $G_C = 3.11 \text{ GeV}^{-2}$.

We neglect the gluon degrees of freedom, especially their fluctuation, which is known to make the CS phase transition first order in the weak coupling region.¹⁾ However, nothing definite is known on the characteristics of the CS in the intermediate density region. In this work, simply assuming that the fluctuation of the pair field dominates that of the gluon field, we examine the effects of the precursory fluctuations of the diquark pair field on the quark sector in the T-matrix approximation (T-approximation).

The DOS $N(\omega)$ is given by

$$N(\omega) = 4 \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \text{Tr}_{c,f} [\rho_0(\mathbf{k}, \omega)], \quad (2.2)$$

where $\rho_0 = (1/4) \text{Tr}[\gamma_0 \mathcal{A}]$ with $\mathcal{A}(\mathbf{k}, \omega) = -1/\pi \cdot \text{Im} G^R(\mathbf{k}, \omega)$ denoting the spectral function of a single quark. The retarded Green function G^R is given by the analytic continuation of the imaginary-time Green function \mathcal{G} , which obeys the following Dyson-Schwinger equation

$$\mathcal{G}(\mathbf{k}, \omega_n) = \mathcal{G}_0(\mathbf{k}, \omega_n) \{1 + \tilde{\Sigma}(\mathbf{k}, \omega_n) \mathcal{G}(\mathbf{k}, \omega_n)\}, \quad (2.3)$$

where $\mathcal{G}_0(\mathbf{k}, \omega_n) = [(i\omega_n + \mu)\gamma^0 - \mathbf{k} \cdot \boldsymbol{\gamma}]^{-1}$ and $\tilde{\Sigma}(\mathbf{k}, \omega_n)$ denote the free Green function and the self-energy in the imaginary time with $\omega_n = (2n + 1)\pi T$.

As was shown in 3), the fluctuating diquark pair field develops a collective mode (the *soft mode* of the CS) at T above but in the vicinity of T_c , in accordance with the Thouless criterion.⁶⁾ Our point in this work is that the soft mode in turn contributes to the self-energy of the quark field, thereby can modify the DOS so much to give rise to a pseudogap.

The quark self-energy $\tilde{\Sigma}$ owing to the soft mode may be obtained by the infinite series of the ring diagrams shown in Fig. 1;

$$\tilde{\Sigma}(\mathbf{k}, \omega_n) = T \sum_{n_1} \int \frac{d^3 \mathbf{k}_1}{(2\pi)^3} \tilde{\Xi}(\mathbf{k} + \mathbf{k}_1, \omega_n + \omega_{n_1}) \mathcal{G}_0(\mathbf{k}_1, \omega_{n_1}), \quad (2.4)$$

$$\tilde{\Xi}(\mathbf{k}, \nu_n) = -8G_C (1 + G_C \mathcal{Q}(\mathbf{k}, \nu_n))^{-1}, \quad (2.5)$$

with the lowest particle-particle correlation function $\mathcal{Q}(\mathbf{k}, \nu_n)$ ³⁾ and $\nu_n = 2n\pi T$.

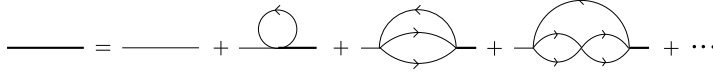


Fig. 1. The Feynman diagrams representing the quark Green function.

Inserting Eqs. (2.4) and (2.5) into Eq. (2.3) and performing the analytic continuation to the upper half of the complex energy plane, we obtain the retarded Green function, $G^R(\mathbf{k}, \omega) = (G_0^{-1}(\mathbf{k}, \omega + i\eta) - \Sigma^R(\mathbf{k}, \omega))^{-1}$, with $\Sigma^R(\mathbf{k}, \omega) = \tilde{\Sigma}(\mathbf{k}, \omega_n)|_{i\omega_n = \omega + i\eta}$. Here, the self-energy Σ^R has the matrix structure $\Sigma^R(\mathbf{k}, \omega) = \Sigma_0(\mathbf{k}, \omega)\gamma^0 - \Sigma_v(\mathbf{k}, \omega)\hat{\mathbf{k}} \cdot \boldsymbol{\gamma} \equiv \gamma^0(\Sigma_- \Lambda_- + \Sigma_+ \Lambda_+)$, where $\Lambda_{\mp} = (1 \pm \gamma^0 \boldsymbol{\gamma} \cdot \hat{\mathbf{k}})/2$ denotes the projection operators onto the positive and negative energy states. $\Sigma_{\mp} = \Sigma_0 \mp \Sigma_v$ represents the self-energies of the particles and anti-particles, respectively.

§3. Numerical Results and Discussions

Since $\rho_0(\mathbf{k}, \omega)$ for $\omega > -\mu$ is well approximated solely by the positive-energy part, we see the characteristic properties of the quark self-energy Σ_- . Numerical calculation shows that $\text{Re}\Sigma_-$ shows a rapid increase around the Fermi energy $\omega = 0$ at $k = k_F$. The behavior of $\text{Re}\Sigma_-$ is responsible for the quark dispersion relation $\omega = \omega(k)$. We show a typical behavior of the quark dispersion relation at $\mu = 400\text{MeV}$ and the reduced temperature $\varepsilon \equiv (T - T_c)/T_c = 0.01$ in Fig. 2. One sees a rapid increase of $\omega(k)$ around the Fermi momentum $k = 400\text{MeV}$, and hence $\partial\omega(k)/\partial k$ becomes large around this momentum. Therefore, the density of states proportional to $(\partial\omega_-/\partial k)^{-1}$ becomes smaller near the Fermi surface, which suggests the existence of a pseudogap. Numerical calculation also shows that there is a peak of $|\text{Im}\Sigma_-|$ around the Fermi energy, which implies that the quasiparticles around this energy are dumped modes. $\text{Im}\Sigma_-$ describes a decay process of a quark to a hole and a diquark, $q \rightarrow h + (qq)$, and this process is enhanced around $\omega = 0$.

The spectral function $\rho_0(\mathbf{k}, \omega)$ is shown in the left panel of Fig. 3, at the same μ and ε as those in Fig. 2. One can see the quasiparticle peaks of the quarks and anti-quarks at $\omega = \omega_-(k) \approx k - \mu$ and $\omega = -k - \mu$, respectively. Notice that the

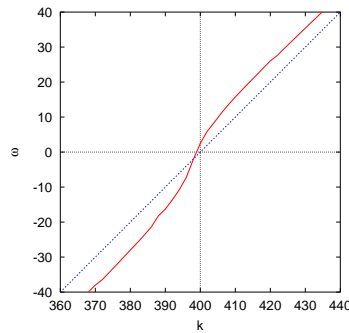


Fig. 2. The quark dispersion relation $\omega = \omega(k)$. One observes a rapid increase of $\omega(k)$ around the Fermi momentum $k = 400\text{MeV}$.

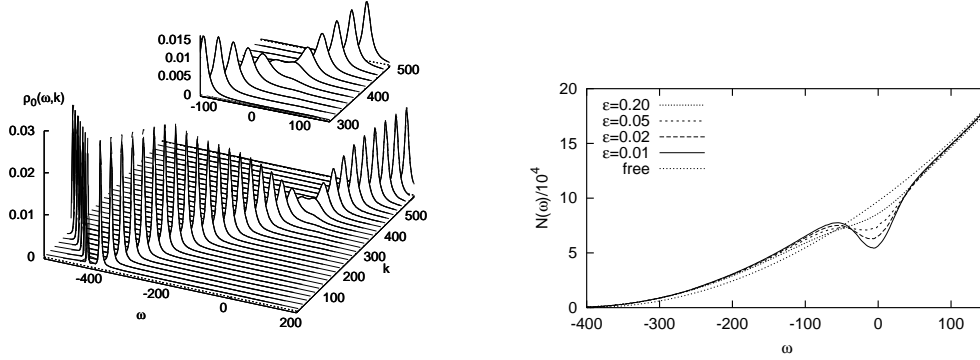


Fig. 3. **Left panel:** The spectral function ρ_0 . There is a depression around $\omega = 0$, which is responsible for the pseudogap formation. **Right panel:** Density of state at $\mu = 400\text{MeV}$ and various $\varepsilon \equiv (T - T_c)/T_c$. A clear pseudogap structure is seen, which survives up to $\varepsilon \approx 0.05$.

quasiparticle peak has a clear depression around $\omega = 0$. The mechanism for the depression is easily understood in terms of the characteristic properties of $\text{Im}\Sigma_-$ mentioned above.

Integrating ρ_0 , one obtains the DOS $N(\omega)$: the right panel of Fig. 3 shows the DOS at $\mu = 400\text{MeV}$ and various values of the reduced temperature ε together with that of the free quark system, $N_0(\omega)$. As anticipated, one can see a remarkable depression of $N(\omega)$, i.e., the *pseudogap*, around the Fermi energy $\omega = 0$; $N(\omega)/N_0(\omega)|_{\omega=0} \simeq 0.55$ at $\varepsilon = 0.01$. The clear pseudogap structure survives even at $\varepsilon = 0.05$. One may thus conclude that there is a pseudogap region within the QGP phase above T_c up to $T = (1.05 \sim 1.1)T_c$ at $\mu = 400\text{MeV}$, for instance. This wide range of T may be just a reflection of the strong coupling nature of the QCD at intermediate density region. Our result obtained for a three-dimensional system tells us that a considerable pseudogap can be formed without the help of the low-dimensionality as in the HTSC and that the pseudogap phenomena in general may be universal in any strong coupling superconductivity.

In this work, we have found that the pseudogap can be formed as a precursory phenomenon of the CS in a rather wide region of T above T_c . It should be noted that our work is the first calculation to show the formation of the pseudogap in the relativistic framework.

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